Statistics
Winter 2022
Lecture 8

Review of odds:

Given $P(E) = .18$

1) Find $P(\bar{E})$

$P(\bar{E}) = 1 - P(E) = 1 - .18 = \boxed{.82}$

2) Find odds in favor of event E.

$P(E) : P(\bar{E}) \Rightarrow .18 : .82 \Rightarrow \boxed{9:41}$

To Simplify .18 ÷ .82 MATH 1: Enter $\frac{9}{41}$

3) Find odds against event E.

$\boxed{41:9}$

Ex:

Odds for event E are $\underset{a}{3} : \underset{b}{32}$

1) Find odds against event E.

$\boxed{32:3}$

2) Find $P(E)$

$P(E) = \frac{a}{a+b} = \boxed{\frac{3}{35}}$

3) Find $P(\bar{E})$

$P(\bar{E}) = \frac{b}{a+b} = \boxed{\frac{32}{35}}$

A box has 40 balls.
25 Red, 10 white, 5 Blue.
Randomly draw 1 ball

1) $P(\text{get Red}) = \frac{25}{40} = \boxed{\frac{5}{8}} = \boxed{.625}$

2) $P(\text{get } \overline{\text{Blue}}) = \frac{35}{40} = \boxed{\frac{7}{8}} = \boxed{.875}$

3) odds in favor of Blue ball $\boxed{1:7}$ ← $\#\text{Blue} : \#\overline{\text{Blue}}$ $5:35$

4) odds against white ball $\#\overline{\text{white}} : \#\text{white}$ $30:10$ → $\boxed{3:1}$

5) odds in favor of Blue OR Red.

$\#(\underbrace{\text{Blue or Red}}_{30}) : \#\overline{(\underbrace{\text{Blue or Red}}_{10})}$

$30:10 \rightarrow \boxed{3:1}$

$P(A) = .72$ $P(B) = .18$ A and B are mutually exclusive events.

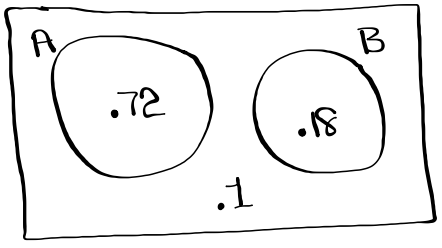
1) $P(\bar{A}) = 1 - P(A)$
$= .28$

2) $P(\bar{B}) = 1 - P(B)$
$= .82$

3) $P(A \text{ and } B) = 0$

4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .72 + .18 - 0 = .9$

5) Construct Venn Diagram.



Given
$P(A) = .65$, $P(B) = .4$ A and B are independent events.

1) $P(\bar{A}) = 1 - .65$
$= .35$

2) $P(\bar{B}) = 1 - .4 = .6$

3) $P(A \text{ and } B)$
$= P(A) \cdot P(B)$
$= (.65)(.4) = .26$

4) $P(A \text{ or } B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= .65 + .4 - .26$
$= .79$

5) Construct Venn Diagram

$P(A \text{ only}) = .65 - .26 = .39$
$P(B \text{ only}) = .4 - .26 = .14$
Total = 1 ✓

6) $P(A \text{ only} \text{ or } B \text{ only}) = .39 + .14 = .53$

7) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - .79 = .21$

De Morgan's Law

8) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - .26 = .74$

A piggy bank has 2 quarters and 10 nickels

Suppose we shake it to drop 2 coins.

Sample Space

NN → 10¢

NQ, QN → 30¢

QQ → 50¢

$$P(\text{Total}=10¢)=P(NN)=\frac{10}{12}\cdot\frac{9}{11}=\frac{90}{132}$$

$$P(\text{Total}=30¢)=P(1N\&1Q)=2\left(\frac{10}{12}\cdot\frac{2}{11}\right)=\frac{40}{132}$$

$$P(\text{Total}=50¢)=P(QQ)=\frac{2}{12}\cdot\frac{1}{11}=\frac{2}{132}$$

Total(¢)	P(Total ¢)
10¢	90/132
30¢	40/132
50¢	2/132

Clear all lists

2nd [+] 4:clear all lists Enter

Reset all lists

STAT Edit 5:Setup editor Enter

Total → L1

P(Total) → L2

L1	L2
10	90÷132 enter
30	40÷132 enter
50	2÷132 enter

STAT → CALC

1: 1-Var Stats

With Menu

List: L1

FreqList: L2

Calculate

No Menu

1-Var Stats

L1, L2

Enter

$\bar{x}=16.\bar{6}$ ✓

S= Blank ✓

$n=1$ ✓

5 Females

10 Males

Select 3 different people

$P(3\text{ Females}) = \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13} = \boxed{\frac{2}{91}}$

FFF
FFM
FMF
FMM
MFF
MFM
MMF
MMM

} Sample Space

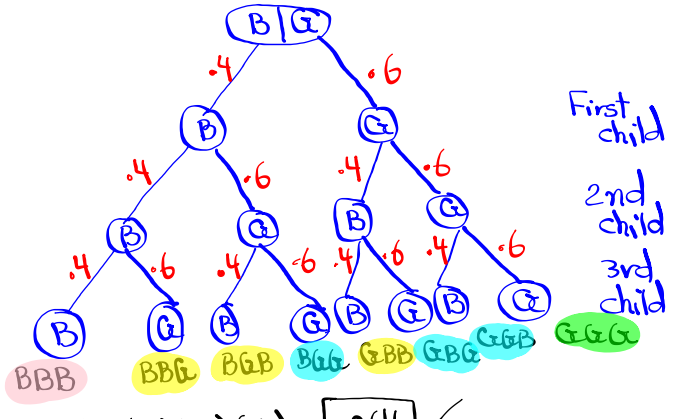
$P(3\text{ Males}) = \frac{10}{15}\cdot\frac{9}{14}\cdot\frac{8}{13} = \boxed{\frac{24}{91}}$

$P(\text{at least 1 Female}) = 1 - P(\text{No Female})$
$= 1 - P(\text{All males}) = 1 - \frac{24}{91} = \boxed{\frac{67}{91}}$

$P(\text{at least 1 Male}) = 1 - P(\text{No males})$
$= 1 - P(\text{All Females}) = 1 - \frac{2}{91} = \boxed{\frac{89}{91}}$

Suppose $P(\text{Girl}) = .6$ $P(\text{Boy}) = .4$

Consider a family with 3 children



$P(\text{3 Boys}) = (.4)(.4)(.4) = \boxed{.064}$ ✓

$P(\text{exactly 2 Boys}) = 3((.4)(.4)(.6)) = \boxed{.288}$ ✓

$P(\text{exactly 1 Boy}) = 3((.4)(.6)(.6)) = \boxed{.432}$ ✓

$P(\text{No Boys}) = (.6)(.6)(.6) = \boxed{.216}$ ✓

# Boys	P(# Boys)
3	.064
2	.288
1	.432
0	.216

Clear all lists

#Boys → L1

P(#Boys) → L2

Use 1-Var Stats with L1 & L2, find

$\bar{x} = 1.2$

S = Blank

n = 1 & Total Prob.

Conditional Prob.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Ex:

$P(A) = .7$

$P(B) = .4$

$P(A \text{ and } B) = .3$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{.3}{.7} = \boxed{\frac{3}{7}} = \boxed{.429}$$

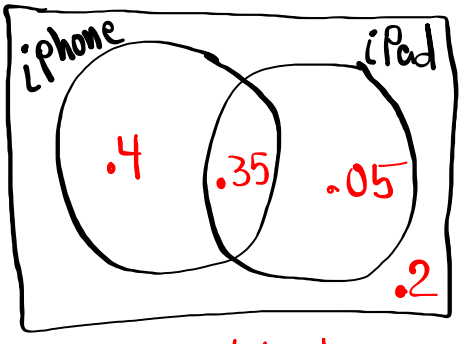
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.3}{.4} = \frac{3}{4}$$

$$= \boxed{.75}$$

$P(\text{iPhone}) = .75$

$P(\text{iPad}) = .4$

$P(\text{iPhone and iPad}) = .35$



Total = 1

$$P(\text{iPad} \mid \text{iPhone}) = \frac{P(\text{iphone and iPad})}{P(\text{iPhone})} = \frac{.35}{.75} = .467 = \frac{7}{15}$$

$$P(\text{iPhone} \mid \text{iPad}) = \frac{P(\text{iphone and iPad})}{P(\text{iPad})} = \frac{.35}{.4} = .875 = \frac{7}{8}$$

Jose is gone shopping.

$P(\text{Pants}) = .55$

$P(\text{shoes}) = .4$

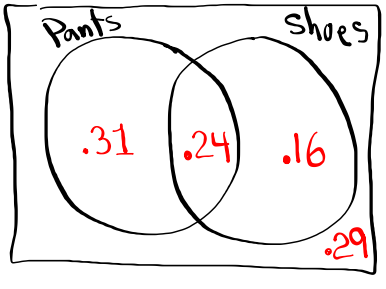
$P(\text{Pants} \mid \text{shoes}) = .6$

$$P(\text{Pants} \mid \text{Shoes}) = \frac{P(\text{Pants and Shoes})}{P(\text{Shoes})}$$

$$.6 = \frac{P(\text{Pants and Shoes})}{.4}$$

Cross-multiply

$P(\text{Pants and shoes}) = (.4)(.6) = .24$



Total = 1 ✓

$$P(\text{shoes} \mid \text{Pants}) = \frac{.24}{.55} = \frac{24}{55} = .436$$

Counting Methods:

Pick a number from 0 to 9.

You have 10 choices

$P(\text{I guess it correctly}) = \frac{1}{10}$

Pick a number from 1 to 25, then Pick a letter from English alphabet.

You have 25 choices for number, then with everyone of those choices, you pick a letter

1A 1B 1C - - - - 1Z
2A 2B 2C - - - - 2Z
⋮
25A 25B - - - - - - 25Z

You have $25(26) = 650$ choices

$P(\text{I guess it correctly}) = \frac{1}{650}$

odds to guess correctly 1:649

5 Students

Adam Bill Carol David Emily

I need to select 2 people

~~AB~~ ~~AC~~ ~~AD~~ ~~AE~~
BA ~~BC~~ ~~BD~~ ~~BE~~
CA CB ~~CD~~ ~~CE~~
DA DB DC ~~DE~~
EA EB EC ED

$5 \cdot 4 = 20$ Selection

If order does not matter, 10 choices

Combination Formula

n different objects, choose r objects, order does not matter

$${}_nC_r = \frac{n!}{r! \cdot (n-r)!}$$

$${}_5C_2 = \frac{5!}{2! \cdot (5-2)!} = 10$$

5 MATH PRB ↓ $_nC_r$ 2 Enter

8 people,
Select 3 of them
in any order
How many ways can this be done?

$$_8C_3 = \boxed{56}$$

CA Lotto → No Mega → Select 5 numbers out of 50 numbers. How many ways?

$$_{50}C_5 = 2,118,760$$

5 Females, 10 Males, Select 3 people, No replacement, order does not matter.

1) How many ways can we do this?

$$_{15}C_3 = 455$$

2) How manys can we select 3 Females?

$$_5C_3 \cdot {}_{10}C_0 = 10$$

3) $P(\text{Select 3 Females}) = \frac{{}_5C_3 \cdot {}_{10}C_0}{{}_{15}C_3} = \frac{10}{455} = \boxed{\frac{2}{91}}$

4) How many was can we Select 3 Males?

$$_5C_0 \cdot {}_{10}C_3 = 120$$

5) $P(\text{Select 3 Males}) = \frac{{}_5C_0 \cdot {}_{10}C_3}{{}_{15}C_3} = \frac{120}{455} = \boxed{\frac{24}{91}}$

2 Quarters,
10 Nickels
Select 2 Coins
No replacement
order does not matter

1) How many ways can we Select 2 coins?

$$12C_2 = \boxed{66}$$

2) How many ways can we select 1 of each?

$$2C_1 \cdot 10C_1 = 20$$

3) $P(1N \& 1Q) = \dfrac{2C_1 \cdot 10C_1}{12C_2} = \dfrac{20}{66} = \dfrac{10}{33}$

A box has 8 Red, 7 white, and 5 Blue color balls.

Select 3 balls.

$$P(1 \text{ of each color}) = \frac{8C_1 \cdot 7C_1 \cdot 5C_1}{20C_3} = \frac{280}{1140} = \boxed{\frac{14}{57}}$$

$$P(2 \text{ Red and } 1 \text{ Blue}) = \frac{8C_2 \cdot 7C_0 \cdot 5C_1}{20C_3} = \frac{140}{1140} = \boxed{\frac{7}{57}}$$

Standard deck of playing cards

52 cards, 4 Aces.

Draw 3 cards, No replacement.

$$P(\text{All aces}) = \frac{{}_4C_3 \cdot {}_{48}C_0}{{}_{52}C_3} = \frac{1}{5525}$$

$$P(\text{exactly 2 Aces}) = \frac{{}_4C_2 \cdot {}_{48}C_1}{{}_{52}C_3} = \frac{12}{5525}$$

$$P(\text{exactly 1 Ace}) = \frac{{}_4C_1 \cdot {}_{48}C_2}{{}_{52}C_3} = \frac{1128}{5525}$$

$$P(\text{No Aces}) = \frac{{}_4C_0 \cdot {}_{48}C_3}{{}_{52}C_3} = \frac{4324}{5525}$$

A company hires 20 people.

10 morning, 8 Afternoon, 2 graveyard shift.

You are the manager to schedule them.

How many ways can you do it?

Morning Afternoon Graveyard

$${}_{20}C_{10} \cdot {}_{10}C_8 \cdot {}_2C_2 = \square$$

$$8,314,020$$

Afternoon Graveyard Morning

$${}_{20}C_8 \cdot {}_{12}C_2 \cdot {}_{10}C_{10} = 8,314,020$$

Live QZ 2

L1	L2
1	.05
2	.15
3	.2
4	.25
5	.25
6	.1

Use L1 & L2 to Find

$\bar{x} = 3.8$

$S =$ Blank

$n = 1$